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Performance Analysis of Reed-Solomon Codes Concatenated with Convolutional Codes over AWGN Channel

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Abstract

With rapid growth in today's technology, digital communication is playing a major role to provide hostile environment to meet various applications. In this communication, Coding plays a prominent role to contribute error free transmission through channel coding which improves capacity of a channel by adding some redundant bit to the original information. One way to provide a better performance of the communication system is by concatenating different types of channel coding techniques. The concatenation can be done either in parallel or serial. The primary aim of this paper is to concatenate the Reed-Solomon codes with Convolutional codes in series, which provides better results comparing with single coding techniques. The performance of the concatenation of Reed-Solomon codes with Convolutional codes can be evaluated by finding bit error rate with various values of signal-to-noise ratio over AWGN channel. The analytical result has been obtained by using MATLAB/OCTAVE.

***Keywords**: Communication System, Convolutional codes, Reed-Solomon codes, Concatenation, signal-to-noise ratio and bit error rate.*

1. Introduction

The function of the communication system is to transfer information from one source (or point) to another source called destination, through a communication channel. The purpose of the transmitter is to transform the message signal produced by the source of information into a form suitable for transmission over the channel. The receiver has the task of operating on the received signal so as to reconstruct a recognizable form of the original message signal and to deliver it to the user destination. Channels of the communication are inevitably noisy and occasionally flip bits that they are transmitting. Here the sender encoders the data being sent by adding redundant bits, and receiver decodes the received string attempting to discover the location of the errors and correcting the errors. The field of channel coding is concerned with sending a stream of data at the highest possible rate over a given communication channel and then decoding the original data reliably at the receiver using encoding and decoding algorithms. In coding theory, concatenation codes form a class of error correcting codes used to achieve a low bit error rate.

2. Reed-Solomon Codes

Reed–Solomon codes were developed in 1960 by Irving S. Reed and Gustavo Solomon, who were then staff members of MITL Lincoln Laboratory. This was entitled "Polynomial Codes over Certain Finite Fields" as it is based on finite field theory.

Reed-Solomon codes are examples of error correcting codes, in which redundant information is added to data so that it can be recovered reliably despite errors in transmission or storage and retrieval. Reed Solomon (R-S) codes form an important sub-class of the family of Bose- Chaudhuri-Hocquenghem (BCH) codes. These are very powerful linear non-binary block codes capable of correcting multiple random as well as burst errors.

2.1. Reed-Solomon Encoding

A Reed-Solomon code is specified as RS (n,k) with s-bit symbols. This means that the encoder takes k data symbols of s bits each and adds parity symbols to make an n symbol codeword. There are n-k parity symbols of s bits each. The Reed-Solomon decoder processes each block and attempts to correct

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errors and recover the original data and can correct up to t symbols that contain errors in a codeword, where $2t = n-k$.

The following diagram shows a typical Reed-Solomon codeword

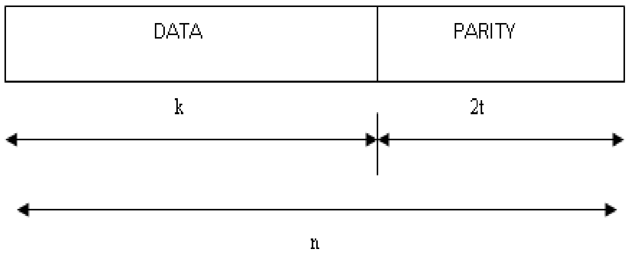

Figure 1. Systematic format of Reed-Solomon codeword

Algorithm for Reed Solomon Encoding:

a. Define the function f(X) or polynomial order 'm'.
b. Check whether the function is a primitive polynomial.
c. If it is a primitive polynomial then calculate Galois field 2^m addition & multiplication for (n,k) block code where

$n = 2^m - 1$
$k = 2^m - 1 - 2t$
t = error correcting capability

d. Generate the generator polynomial G(X) depending upon the no. of roots

i.e. $G(X) = g_0 + g_1 X + g_2 X^2 + \ldots + X^{2t}$

e. To generate parity check matrix P(X), multiply the message polynomial m(X) with n-k and divide by generator polynomial G(X).
f. Then the encoded codeword U(X) is generated by combing $m(X).X^{n-k}$ with P(X).

i.e. $U(X) = m(X).X^{n-k} + P(X)$

2.2. Reed-Solomon Decoding

Reed-Solomon error correcting codes are frequently used in communication systems and data storage, to get back data from possible errors that take place throughout transmission. One unique application of the Reed-Solomon codes is the forward error correction (FEC) i.e., the encoder attaches parity symbols to the data using a predetermined algorithm before transmission. When a received block is input to the decoder for processing, the decoder first verifies whether this block appears in the dictionary of valid code words. If it does not, errors must have occurred during transmission. This part of the decoder processing is called error detection. The parameters which are necessary to reconstruct the original encoded block are available to the decoder. If errors are detected, the decoder attempts a reconstruction. This is called error correction.

Algorithm for Reed-Solomon Decoding:

a. The received code word r(X) is the original code word U(X) plus error code word e(X)
   i.e., r(X) = U(X)+e(X)
b. The received code word has 2t syndromes that depends only on errors and can be calculated by substituting 2t roots of g(X) into r(X).

c. Error locator polynomial is obtained by using Berlekamp-Massey algorithm containing‘t’ unknowns.
d. Obtain the roots of the polynomial by using chien search algorithm i.e., to locate the error positions or locations.
e. The magnitudes of the error locations are obtained by using Forney algorithm.
f. Thus Corrected codeword is obtained as c(X).

3. Interleaver and Deinterleaver

The performance of Forward Error Correction (FEC) systems operating in the presence of burst errors is improved by passing the coded signal through an interleaver. The interleaving process is added between two codes to spread burst errors across a wider range. Interleaving is a technique commonly used in communication systems to overcome correlated channel noise such as burst error or fading. The interleaver rearranges input data such that consecutive data are spaced apart. Interleaving is a process or methodology to make a system more efficient, fast and reliable by arranging data in a noncontiguous manner. At the receiver end, the interleaved data is arranged back into the original sequence by the deinterleaver. As a result of interleaving, correlated noise introduced in the transmission channel appears to be statistically independent at the receiver and thus allows better error correction.

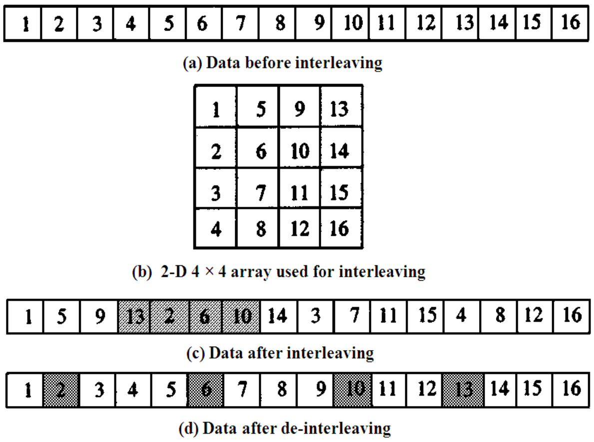

Figure 2. Example for interleaving technique

4. Convolutional Codes

Provide a statement that what is expected, as stated in the "Introduction" chapter can ultimately result in "Results and Discussion" chapter, so there is compatibility. Moreover, it can also be added the prospect of the development of research results and application prospects of further studies into the next (based on result and discussion).

4.1. Convolutional Encoding

Convolutional codes accept a continuous stream of bits and map them into an output stream introducing redundancies in the process. The efficiency or data rate of a Convolutional code is measured by the ratio of the number of bits in the input k to the number of bits in the output n.

In a Convolutional code, there is some ‘memory’ that remembers the stream of bits flow by which is used to encode the following bits. Code rate ‘r’ is determined by input rate and output rate

$$r = \frac{r_{input}}{r_{output}} < 1$$

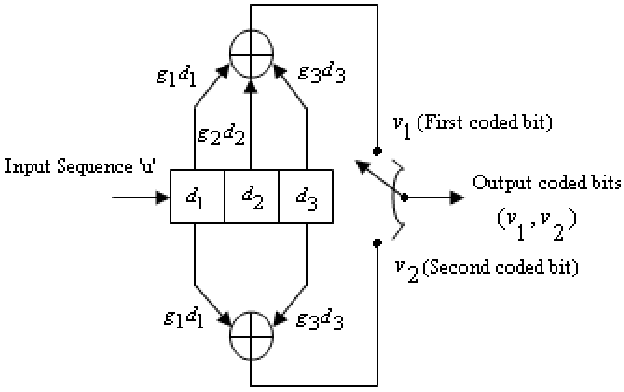

Figure 3. Convolutional Encoding

Algorithm For Convolutional Encoding:

a. Convolutional codes are generated using generator polynomials which are multiplied with input sequence and then XORed.
b. Let the inputs be $u_1(X), u_2(X), \ldots\ldots\ldots. u_n(X)$.
c. Let the outputs be $v_1(X), v_2(X), \ldots\ldots\ldots v_n(X)$
d. Where 'v' is called a code word.
e. When message sequence enter the shift register bit by bit, it gets multiplied withgenerated polynomial and is XORed using Modulo-2 adders to obtain the expressions

$$v_1(X) = g_1(X)d_1 + g_2(X)d_2 + g_3(X)d_3$$

$$v_2(X) = g_1(X)d_1 + g_3(X)d_3$$

f. Where, d_1, d_2, d_3 are shown in Figure 3.
g. Above procedure is repeated for 'n' number of input sequence bits.

4.2. Convolutional Decoding

Errors occur during transmission or storage for a number of reasons like noise, interference, etc. Convolutional decoder processes sequence of encoded bits and corrects errors and recover the original data.

One of the decoding algorithms is Viterbi algorithm which is published by Viterbi in 1967. The algorithm is optimal in the maximum-likelihood sense, and has quickly become the most widely used Convolutional decoding algorithm in practice for its reduced computational complexity and satisfactory performance. The Viterbi algorithm is a maximum likelihood decoder, meaning that the output code word from decoding a transmission is always the one with the highest probability of being the correct word transmitted from the source.

Algorithm for Convolutional Decoding:

a. Let the binary data is given as input to the convolutional decoder.
b. Select an input pair and calculate the branch metric value i.e., the hamming distance between the input pair and ideal pairs ("00","01","10","11").
c. Then choose the path for which path metric value is minimum and thus obtain the decoded codeword.

5. Concatenation of Reed-Solomon Codes With Convolutional Codes

Forney (1966) showed that concatenated codes could be used to achieve exponentially decreasing error probabilities at all data rates less than capacity. This also includes the decoding complexity that increases only polynomially with the code block length N.

The basic concatenated coding scheme is shown in Figure 4. Forney proposed to construct a channel coding system by concatenating two codes, an outer code and an inner code. Both encoders are separated by an interleaver, whereas the related decoders are separated by a dual block to the interleaver, i.e. by a deinterleaver.

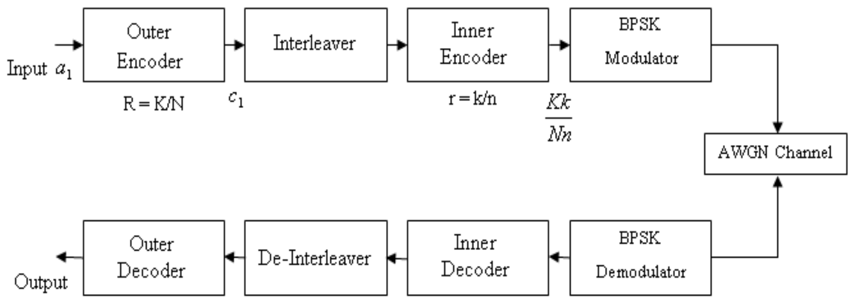

Figure 4. Block diagram of Concatenation technique

In the figure, Input message blocks as vectors are coded in the outer encoder of the code rate R = K/N. Code words are obtained. Subsequently, bits of code words are interleaved. As a result, the received output block consists of the input bits whose time sequence is changed with respect to the input. The resulting binary block constitutes the input sequence of the inner code encoder with the code rate equal to r = k/n. As we can see, the code rate of the code concatenation is in fact $\frac{Kk}{Nn}$.

The receiver performs the operations that are dual to those made in the transmitter. Additive white Gaussian noise (AWGN) is common to every communication channel which is the statistically random radio noise characterized by a wide frequency range with regards to a signal in the communication channel. AWGN is a channel model in which the only impairment to communication is a linear addition of wideband or white noise with a constant spectral density (expressed as watts per hertz of bandwidth) and a Gaussian distribution of amplitude. Thus, the decoder of the inner code decides upon the input message sequence on the basis of the sequence corrupted by the channel and noise. The resulting sequence is a subject of de-interleaving in which the original time sequence of message bits is recovered. Finally, the outer decoder decodes the de- interleaved sequence.

6. Results

Concatenation is a method of combining two codes such as Reed-Solomon codes and convolutional codes in order to achieve high bit error rate, to minimize the burst errors.

First we performed the simulations for RS-CC codes for different block lengths. We can see from Figure 5, as the block length increases the BER performance improves. Form the Figure.5 we can also see that the RS-CC (255,165) gives best results with m=8; i.e. number of bits per symbol is 8.

Secondly we performed the simulations BER vs SNR for concatenated codes and single codes. Figure 6 shows the simulation results. From Figure 6 it can be seen that as SNR increases, bit error rate decreases.

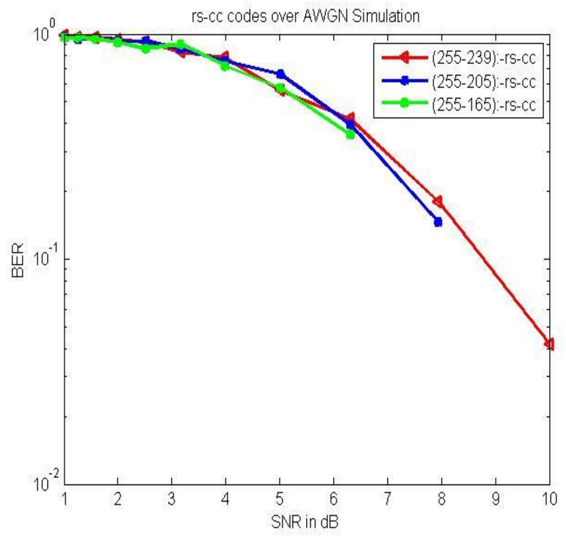

Figure 5. The BER performance of different RS-CC codes

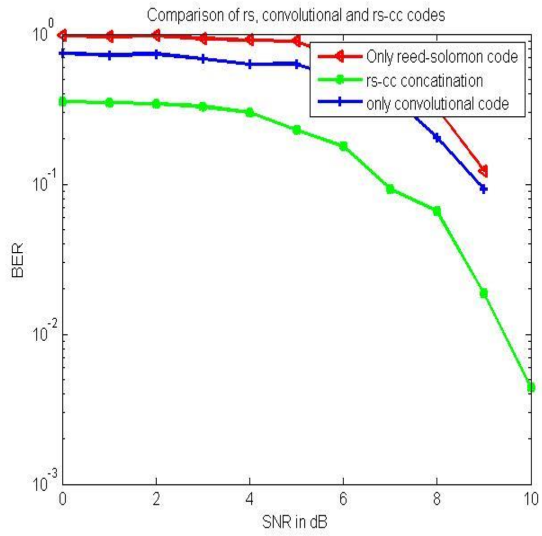

Figure 6. The BER performance of RS-CC codes with single codes.

7. Conclusion and Future Scope

This paper gives deep and clear understanding of Reed-Solomon codes and Convolutional codes making them simpler and easier to understand and implement. From fig.6 we can conclude that bit error performance of rs-cc codes is good comparing with the single codes. In this paper we discussed the serial concatenation of error correcting codes. We can also perform parallel concatenation of two or more error correcting codes. We will observe the error performance of both the concatenated codes.

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